CCSF PHYC 4D Lecture Notes

Karl M. Westerberg

Chapter 1
Introduction

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444 Castro Street, Suite 900
Mountain View, CA 94041
USA

Physics review

Students should read this chapter and review material from PHYC 4A, 4B, and 4C. We will be revisiting this material from time to time during the semester.

Prefixes and units

Prefix	Symbol	Factor
deca-	da	10^1
hecto-	h	10^2
kilo-	k	10^3
mega-	M	10^6
giga-	G	10^9
tera-	T	10^{12}
peta-	P	10^{15}
exa-	E	10^{18}
zetta-	Z	10^{21}
yotta-	Y	10^{24}

Prefix	Symbol	Factor
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}
atto-	a	10^{-18}
zepto-	z	10^{-21}
yocto-	y	10^{-24}

$$1\,\text{eV} = 1.602177 \times 10^{-19}\,\text{J}; \qquad 1\,\text{Å} = 10^{-10}\,\text{m}$$

Special relativity

$$c = 2.997925 \times 10^8\,\text{m/s} = 0.299792\,\text{m/ns} = 0.983571\,\text{ft/ns}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 1 + \frac{1}{2}v^2/c^2 + \frac{3}{8}v^4/c^4 + \dots$$

γ is the "relativistic correction factor", and is approximately equal to 1.005 when $v = 0.1c$, indicating a 0.5% relativistic correction at those speeds.

Rest energy is related to mass via $E = mc^2$. Rest energies for electrons, protons, and neutrons can be expressed in MeV.

$$E_e = 0.510999\,\text{MeV}; \qquad E_p = 938.272\,\text{MeV}; \qquad E_n = 939.565\,\text{MeV}$$

These represent the energies required to create these particles, and the amount of energy released when the particles are annihilated.

Electromagnetic spectrum

In a vacuum, $c = \nu\lambda$ (ν = frequency, λ = wavelength). Approximate wavelength and frequency ranges for different parts of the electromagnetic spectrum are given in Fig. 1.7[1.4] (p. 9[6]) of the textbook (numbers in brackets refer to the old *second* edition of Krane).

For visible light: $380\,\text{nm}\,(\text{blue}) \leq \lambda \leq 750\,\text{nm}\,(\text{red})$ and $790\,\text{THz} \geq \nu \geq 400\,\text{THz}$.

Quantum mechanics

$$h = 6.626070 \times 10^{-34}\,\text{J s} = 4.135668\,\text{eV fs}$$
$$\hbar = h/2\pi = 1.054572 \times 10^{-34}\,\text{J s} = 0.6582119\,\text{eV fs}$$
$$hc = 1.986446 \times 10^{-25}\,\text{J m} = 1239.84\,\text{eV nm} = 1239.84\,\text{MeV fm}$$
$$\hbar c = 3.161526 \times 10^{-26}\,\text{J m} = 197.327\,\text{eV nm} = 197.327\,\text{MeV fm}$$

Photon energies are given by $E = h\nu = hc/\lambda$. Visible light: $3.3\,\text{eV}\,(\text{blue}) \geq E \geq 1.7\,\text{eV}\,(\text{red})$.

Atomic physics

For two singly charged particles ($q = \pm e$),

$$|U| = e^2/(4\pi\epsilon_0 r)$$
$$e^2/(4\pi\epsilon_0) = 1.439964\,\text{eV nm} = 1.439964\,\text{MeV fm}$$

sets the energy scale for electromagnetic interactions. $|U| = 27.2\,\text{eV}$ for $r = 0.529\,\text{Å}$ (atomic scale). $|U| = 1.4\,\text{MeV}$ for $r = 1\,\text{fm}$ (nuclear scale).

The fine structure constant (α) provides a dimensionless measure of the electromagnetic interaction.

$$\alpha = e^2/(4\pi\epsilon_0\hbar c) = 1/137.036$$

The fact that α is much less than one implies that a photon whose wavelength is small enough to "see" an atom is large enough (in energy) to ionize it. It is hard to "see" atoms without profoundly affecting them.

Hydrogen atom radius and energy, as predicted by the Bohr Model and by Schrodinger's equation:

$$a_0 = \hbar^2 4\pi\epsilon_0/(m_e e^2) = 0.5291772\,\text{Å}$$

$$E_0 = \frac{1}{2} e^2/(4\pi\epsilon_0 a_0) = 13.60569\,\mathrm{eV}$$

Statistical mechanics

$$k = 1.380649 \times 10^{-23}\,\mathrm{J/K}$$

At room temperature ($T = 300\,\mathrm{K}$), $kT = 0.026\,\mathrm{eV} \approx \frac{1}{40}\,\mathrm{eV}$. The equipartition theorem of classical physics states that each mode carries an average energy of $\frac{1}{2}kT$. One of the early indications of quantum mechanics is the failure of the equipartition theorem to correctly predict the heat capacities of ideal gasses.

Gravity and general relativity

$$G = 6.673840 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$$

Newton's "Universal Law of Gravitation" (which is actually an approximation to Einstein's General Relativity appropriate for weak gravitational fields) predicts the gravitational potential (analogous to the electric potential) for a spherically symmetric object to be given by

$$\phi = U_g/m = -GM/r = -gr$$

Near the surface of the earth,

$$-\phi = 6.251101 \times 10^7\,\mathrm{J/kg} = 6.955288 \times 10^{-10}\,c^2$$

which is considered "weak field". Strong field occurs when $-\phi$ is (approximately) equal to c^2, or when $r = GM/c^2$.

The event horizon of a black hole occurs at a radius given by

$$r = (2G/c^2)M; \qquad 2G/c^2 = 1.485130 \times 10^{-27}\,\mathrm{m/kg}$$

The event horizon for the earth is less than 1 cm. The event horizan for the sun is 2.95 km. If the mass of the sun can be confined to this radius, the sun would become a black hole. Most objects fail to be confined within their event horizons, and so remain as ordinary objects. After "burning out", however, sufficiently large mass stars will eventually collapse into a black hole.

Planck scale / Quantum gravity

c, G, and $\hbar$ can be combined to form Planck scale units which define the intersection of special relativity, general relativity, and quantum mechanics. Expressed in these units, c, G, and $\hbar$ all equal 1.

$$L_{\text{pl}} = \sqrt{\frac{\hbar G}{c^3}} = 1.616199 \times 10^{-35}\,\text{m}$$

$$T_{\text{pl}} = \sqrt{\frac{\hbar G}{c^5}} = 5.391060 \times 10^{-44}\,\text{s}$$

$$M_{\text{pl}} = \sqrt{\frac{\hbar c}{G}} = 2.176509 \times 10^{-8}\,\text{kg}$$

$$E_{\text{pl}} = M_{\text{pl}} c^2 = 1.956149 \times 10^{9}\,\text{J} = 1.220932 \times 10^{19}\,\text{GeV}$$

We are not anywhere near to producing these energies in experiments, so the study of quantum gravity is almost entirely theoretical at this point. General Relativity and Quantum Mechanics don't mix as they stand, so we know that a new theory must replace them, but what? String theory? Lattice theory? No one can be sure.

The age of the universe is currently estimated to be $13.75 \times 10^{9}\,\text{yr}$ or $4.339 \times 10^{17}\,\text{s}$. The ratio of this time to the Planck time is 8.049×10^{60}. If you believe that spacetime forms a lattice at the Planck scale, then this ratio represents the number of lattice points on each side of the hypercube representing the known universe. That implies that approximately 4.2×10^{243} lattice points exist in the portion of the four-dimensional universe that is observable at this time.